[11] P. Steffen, P. N. Heller, R. A. Gopinath, and C. S. Burrus, "Theory of regular M-band wavelet bases," *IEEE Trans. Signal Processing*, vol. 41, pp. 3497–3511, Dec. 1993.
[12] P. Heller, H. L. Resnikoff, and R. O. Wells, "Wavelet matrices and the representation of discrete functions," in *Wavelets: A Tutorial in Theory and Applications*, C. K. Chui, Ed. New York: Academic, 1992.
[13] P. Heller, "The construction of rank M wavelet with N vanishing moments," *SIAM J. Matrix Anal.*, vol. 16, 1995.
[14] E. Kofidis, S. Theodoridis, and N. Kalouptsidis, "On the perfect-reconstruction problem in N-band multirate maximally-decimated FIR filter banks," *IEEE Trans. Signal Processing*, vol. 44, pp. 2439–2455, Oct. 1996.
[15] M. Ikehara and T. Q. Nguyen, "Time domain design of linear-phase PR filter banks," *Proc. ICASSP*, 1997.
[16] T. Q. Nguyen, "Partial spectrum reconstruction using digital filter banks," *IEEE Trans. Signal Processing*, vol. 41, pp. 1128–1139, Mar. 1993.
[17] M. R. Raghuveer, "Wavelets as signaling waveforms over partial-response channels," in *Proc. Coding Signal Process. Inform. Storage*: SPIE, 1995, vol. 2605, pp. 24–28.
[18] T. Q. Nguyen, "Digital filter bank design quadratic-constrained formulation," *IEEE Trans. Signal Processing*, vol. 43, pp. 2103–2108, Sept. 1995.
[19] T. Nagai, C. W. Kok, M. Ikehara, and T. Q. Nguyen, "Design and lattice structure of FIR paraunitary filter banks with linear phase," *IEICE Trans. Fund. Electron., Commun. Comput. Sci.*, pp. 712–721, Apr. 1997.

Some Properties of Symmetric–Antisymmetric Orthonormal Multiwavelets

Lixin Shen, Hwee Huat Tan, and Jo Yew Tham

***Abstract*—We analyze the discrete multiwavelet transform using symmetric–antisymmetric orthonormal multifilters (SAOMF's) and prove that for any even-length SAOMF, we can always find an odd-length SAOMF such that the implementation of discrete multiwavelet transform using either the even-length or the odd-length SAOMF produces identical output for a given input signal if the sum/difference prefilter is chosen.**

***Index Terms*—Multifilters, multiwavelets, prefilter.**

I. Introduction

Multiwavelets are generated by more than one scaling function. There are many degrees of freedom in the construction of multiwavelets. These allow for more features to be built into a multiwavelet transform and enable one to construct multiwavelet filters to suit one's needs. For instance, a multiwavelet can have small support, orthonormal integer translates, as well as symmetry [2]. However, as is known, there exist some important differences between multiwavelet and scalar wavelet bases, and these differences become apparent when one implements the discrete multiwavelet transform (DMWT). First, in the processing of discrete-time signals, preprocessing (prefiltering) of the discrete data is an essential and necessary step that does not arise for scalar wavelet transforms. Several authors have addressed the design of prefilters that are specifically suited for the DMWT [3], [5], [6], [8]–[10], [12], [13]. Second, for multiwavelet bases, one only focuses on the zero moment properties of the associated multifilter banks and the construction of specialized K-balanced or "good frequency characteristics" multiwavelets [4]–[7], [9], [11].

In our previous paper [7], it was shown that the length-$2N$ and length-$(2N+1)$ multifilters, where the associated multiwavelets are symmetric and antisymmetric pairs, can be obtained from one another. However, from a mathematical viewpoint, the corresponding multiwavelets may have different compact support, regularity, and approximation order. We will point out these differences with some examples. This paper highlights the equivalence between even-length and odd-length symmetric–antisymmetric orthonormal multifilters (SAOMF's), which can be derived from one another, for applications in signal processing if the sum/difference prefilter is used.

Manuscript received May 12, 1999; revised January 30, 2000. This work was supported by the Centre for Wavelets, Approximation, and Information Processing (CWAIP), which is funded by the National Science and Technology Board and the Ministry of Education under Grant RP960 601/A and the National University of Singapore. The associate editor coordinating the review of this paper and approving it for publication was Prof. Chin-Liang Wang.

The authors are with the Department of Mathematics, National University of Singapore, Singapore (e-mail: shenlx@wavelets.math.nus.edu.sg; mattanhh@wavelets.math.nus.edu.sg; thamjy@wavelets.math.nus.edu.sg).

Publisher Item Identifier S 1053-587X(00)04926-6.

II. Preliminaries

An orthonormal multiwavelet system with multiplicity r consists of one multiscaling function vector $\boldsymbol{\phi}(x) = [\phi_1(x), \cdots, \phi_r(x)]^T$ and one multiwavelet vector $\boldsymbol{\psi}(x) = [\psi_1(x), \cdots, \psi_r(x)]^T$. $\boldsymbol{\phi}$ generates a multiresolution analysis $\{V_k\}_{k\in\mathbb{Z}}$ of $L^2(\mathbb{R})$. In the following, we will restrict ourselves to considering only multiscaling functions and multiwavelets with compact support. The vectors $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ satisfy the following refinement and wavelet equations:

$$\boldsymbol{\phi}(x) = \sum_{k\in\mathbb{Z}} \boldsymbol{P}_k \boldsymbol{\phi}(2x-k) \tag{1}$$

$$\boldsymbol{\psi}(x) = \sum_{k\in\mathbb{Z}} \boldsymbol{Q}_k \boldsymbol{\phi}(2x-k) \tag{2}$$

respectively, where $\{\boldsymbol{P}_k\}$ and $\{\boldsymbol{Q}_k\}$ are finite $r\times r$ matrix sequences. We will refer to ϕ_i's and ψ_i's as the multiscaling and multiwavelet functions, respectively, and the matrix sequences $\{\boldsymbol{P}_k\}$ and $\{\boldsymbol{Q}_k\}$ as the lowpass and highpass finite impulse responses, respectively.

If a continuous-time signal $v(t) \in V_0$ can be expanded as

$$v(t) = \sum_k \boldsymbol{\phi}^T(t-k)\boldsymbol{v}_k^{(0)}$$

where $\boldsymbol{v}_k^{(0)}$ is the vectorized discrete-time signal representative of the input signal $v(t)$, then it follows that we have the well-known relations

$$\boldsymbol{v}_n^{(1)} = \sum_k \boldsymbol{P}_{k-2n}\boldsymbol{v}_k^{(0)}, \qquad \boldsymbol{d}_n^{(1)} = \sum_k \boldsymbol{Q}_{k-2n}\boldsymbol{v}_k^{(0)} \tag{3}$$

for the analysis stage, and

$$\boldsymbol{v}_n^{(0)} = \sum_k \boldsymbol{P}_{n-2k}^T\boldsymbol{v}_k^{(1)} + \sum_k \boldsymbol{Q}_{n-2k}^T\boldsymbol{d}_k^{(1)} \tag{4}$$

for the synthesis stage.

III. Symmetric–Antisymmetric Orthonormal Multifilters

Several works [1], [4], [5], [7] studied the special case of multiwavelet systems for multiplicity $r=2$, where the filters are SAOMF's. We say that a finite length matrix sequence $\{\boldsymbol{P}_k\}_{k=N^\ell}^{N^u}$ satisfies *Condition SA* if the following condition holds:

$$\boldsymbol{P}_k = \boldsymbol{S}\boldsymbol{P}_{N^u+N^\ell-k}\boldsymbol{S}, \qquad k = N^\ell, \cdots, N^u \tag{5}$$

where $\boldsymbol{S} = \mathrm{diag}(1, -1)$. As shown in [1], (5) is required for symmetric–antisymmetric multiscaling functions, which can be expressed

in the form $\phi_1(x) = \phi_1(N^u + N^\ell - x)$ and $\phi_2(x) = -\phi_2(N^u + N^\ell - x)$. The lowpass sequences $\{\boldsymbol{P}\}_{k=N^\ell}^{N^u}$ that satisfy Condition SA will lead to SAOMF's if other conditions such orthogonality and conjugate quadrature filter conditions are satisfied. For ease of notation, without loss of generality, we will assume that $N^\ell = 0$ in the following presentation.

For any odd-length SAOMF $\{\boldsymbol{P}_k\}_{k=0}^{2N}$, we can always assume that (see [7])

$$\boldsymbol{P}_0 = \begin{bmatrix} \alpha & \alpha \\ \beta & \beta \end{bmatrix} \quad \text{and } \boldsymbol{P}_{2N} = \begin{bmatrix} \alpha & -\alpha \\ -\beta & \beta \end{bmatrix}. \tag{6}$$

Denote

$$\Delta_1 := \tfrac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \quad \text{and } \Delta_2 := \tfrac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}.$$

Theorem 1 ([7]): Let $\{\boldsymbol{P}_{e,k}\}_{k=0}^{2N-1}$ be an even-length SAOMF. Construct

$$\boldsymbol{P}_{o,k} = \boldsymbol{P}_{e,k-1}\boldsymbol{\Delta}_1 + \boldsymbol{P}_{e,k}\boldsymbol{\Delta}_2, \qquad k = 0, \cdots, 2N \tag{7}$$

where $\boldsymbol{P}_{e,-1} = \boldsymbol{0}$, and $\boldsymbol{P}_{e,2N} = \boldsymbol{0}$. Then, $\{\boldsymbol{P}_{o,k}\}_{k=0}^{2N}$ is an odd-length SAOMF. Conversely, given an odd-length SAOMF $\{\boldsymbol{P}_{o,k}\}_{k=0}^{2N}$, construct

$$\boldsymbol{P}_{e,k} = \boldsymbol{P}_{o,k}\boldsymbol{\Delta}_1 + \boldsymbol{P}_{o,k+1}\boldsymbol{\Delta}_2, \qquad k = 0, \cdots, 2N-1. \tag{8}$$

Then, $\{\boldsymbol{P}_{e,k}\}_{k=0}^{2N-1}$ is an even-length SAOMF.

This theorem is also true for the associated highpass multifilters satisfying Condition SA.

The conversion from even-length SAOMF's to odd-length SAOMF's usually increases the compact support and changes the regularity and the approximation order of the corresponding multiscaling function $\boldsymbol{\phi}$. This can be illustrated by the following example.

Example—A Class of Length-4 SAOMF's: A class of length-4 SAOMF's with one parameter is given [7] by

$$\boldsymbol{P}_{e,0} = \begin{bmatrix} \dfrac{1}{\alpha^2+1} & \dfrac{\alpha}{\alpha^2+1} \\ \dfrac{1}{\alpha^2+1} & -\dfrac{\alpha}{\alpha^2+1} \end{bmatrix}$$

$$\boldsymbol{P}_{e,1} = \begin{bmatrix} \dfrac{\alpha^2}{\alpha^2+1} & \dfrac{\alpha}{\alpha^2+1} \\ -\dfrac{\alpha^2}{\alpha^2+1} & \dfrac{\alpha}{\alpha^2+1} \end{bmatrix}$$

and $\boldsymbol{P}_{e,2} = \boldsymbol{S}\boldsymbol{P}_{e,1}\boldsymbol{S}$, $\boldsymbol{P}_{e,3} = \boldsymbol{S}\boldsymbol{P}_{e,0}\boldsymbol{S}$. The associated highpass multifilter is given by $\boldsymbol{Q}_{e,k} = (-1)^{k+1}\boldsymbol{P}_{e,3-k}\boldsymbol{A}$ for $k = 0, 1, 2, 3$, where $\boldsymbol{A} = \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$. The multiscaling functions for this class of SAOMF's are supported on $[0, 3]$. The special case when $\alpha = 4 + \sqrt{19}$ gives the SA4(1) filter, which has approximation order two and a Sobolev exponent of 1.5270 for its multiscaling functions. Another case when $\alpha = 4 + \sqrt{11}$ gives the SA4(2) filter, which has approximation order one and a Sobolev exponent of 0.9650 for its multiscaling functions. By applying Theorem 1, we can derive from the above class of length-4 SAOMF's a class of one-parameter length-5 SAOMF's as

$$\boldsymbol{P}_{o,0} = \begin{bmatrix} \dfrac{1-\alpha}{2(\alpha^2+1)} & \dfrac{1-\alpha}{2(\alpha^2+1)} \\ \dfrac{1+\alpha}{2(\alpha^2+1)} & \dfrac{1+\alpha}{2(\alpha^2+1)} \end{bmatrix}$$

$$\boldsymbol{P}_{o,1} = \begin{bmatrix} \dfrac{1}{2} & \dfrac{\alpha^2-2\alpha-1}{2(\alpha^2+1)} \\ \dfrac{1-2\alpha-\alpha^2}{2(\alpha^2+1)} & -\dfrac{1}{2} \end{bmatrix}$$

$$\boldsymbol{P}_{o,2} = \begin{bmatrix} \dfrac{\alpha+\alpha^2}{\alpha^2+1} & 0 \\ 0 & \dfrac{\alpha^2-\alpha}{\alpha^2+1} \end{bmatrix}$$

$\boldsymbol{P}_{o,3} = \boldsymbol{S}\boldsymbol{P}_{o,1}\boldsymbol{S}$, and $\boldsymbol{P}_{o,4} = \boldsymbol{S}\boldsymbol{P}_{o,0}\boldsymbol{S}$. The multiscaling functions for this class of SAOMF's are supported on $[0, 4]$. Using the same two values of α for obtaining SA4(1) and SA4(2) gives us the SA5(1) and SA5(2) filters, respectively. Now, in contrast to its length-4 "relative," SA5(1) has only approximation order one and a Sobolev exponent of 0.9691 for its multiscaling functions. Furthermore, SA5(2), in contrast to SA4(2), has approximation order two and a Sobolev exponent of 1.4947 for its multiscaling functions. The associated highpass multifilter $\{\boldsymbol{Q}_{o,k}\}_{k=0}^{4}$ can be obtained from $\{\boldsymbol{Q}_{e,k}\}_{k=0}^{3}$ and Theorem 1. See Fig. 1 for plots of multiscaling functions for SA4(1) and SA5(1) filters. Although the function profiles appear identical, there are differences on closer examination.

In the following, we always assume that $\{\boldsymbol{P}_{e,k}\}_{k=0}^{2N-1}$ and $\{\boldsymbol{P}_{o,k}\}_{k=0}^{2N}$ can be derived from one another according to Theorem 1.

IV. Discrete Multiwavelets Transform Using SAOMFs

Although the mathematical properties of $\boldsymbol{\phi}$ may be totally changed when one converts from an even-length SAOMF to an odd-length SAOMF, we will show in this section that for any such pair of related SAOMF's, when the two input vector sequences satisfy certain linear relations, then the same output vector sequence can be produced from the discrete multiwavelet transforms (3). These relations are precisely expressed in the following theorem.

Theorem 2: Let two vector sequences $\{\boldsymbol{v}_k\}$ and $\{\boldsymbol{u}_k\}$ satisfy

$$\boldsymbol{u}_k = \Delta_2\boldsymbol{v}_{k-1} + \Delta_1\boldsymbol{v}_k. \tag{9}$$

Then, for any integer n, we have the following three relationships:

a) $\sum_{k=0}^{2N-1}\boldsymbol{P}_{e,k}\boldsymbol{v}_{k+2n} = \sum_{k=0}^{2N}\boldsymbol{P}_{o,k}\boldsymbol{u}_{k+2n}$.
b) $\sum_{k=0}^{2N}\boldsymbol{P}_{o,k}\boldsymbol{v}_{k+2n} = \sum_{k=0}^{2N-1}\boldsymbol{P}_{e,k}\boldsymbol{u}_{k+1+2n}$.
c) $\sum_{k=0}^{2N-1}\boldsymbol{P}_{e,k}\boldsymbol{u}_{k+2n} = \sum_{k=0}^{2N}\boldsymbol{P}_{o,k}\boldsymbol{v}_{k-1+2n}$.

Proof: We first treat a). From Theorem 1 and the facts, $\boldsymbol{P}_{o,0}\boldsymbol{\Delta}_2 = \boldsymbol{P}_{o,2N}\boldsymbol{\Delta}_1 = \boldsymbol{0}$, and we can conclude that

$$\begin{aligned}\sum_{k=0}^{2N-1}\boldsymbol{P}_{e,k}\boldsymbol{v}_{k+2n} &= \sum_{k=0}^{2N-1}(\boldsymbol{P}_{o,k}\boldsymbol{\Delta}_1 + \boldsymbol{P}_{o,k+1}\boldsymbol{\Delta}_2)\boldsymbol{v}_{k+2n} \\ &= \sum_{k=0}^{2N}\boldsymbol{P}_{o,k}(\boldsymbol{\Delta}_1\boldsymbol{v}_{k+2n} + \boldsymbol{\Delta}_2\boldsymbol{v}_{k-1+2n}) \\ &= \sum_{k=0}^{2N}\boldsymbol{P}_{o,k}\boldsymbol{u}_{k+2n}.\end{aligned}$$

Hence, a) holds. Similarly, we can show that b) holds.

Note that $\boldsymbol{\Delta}_1\boldsymbol{\Delta}_1 = \boldsymbol{0}$, $\boldsymbol{\Delta}_2\boldsymbol{\Delta}_2 = \boldsymbol{0}$, and $\boldsymbol{\Delta}_2\boldsymbol{\Delta}_1 + \boldsymbol{\Delta}_1\boldsymbol{\Delta}_2 = \boldsymbol{I}$. If two vector sequences $\{\boldsymbol{v}_k\}$ and $\{\boldsymbol{u}_k\}$ satisfy (9), then it is easy to show that

$$\boldsymbol{v}_k = \boldsymbol{\Delta}_2\boldsymbol{u}_k + \boldsymbol{\Delta}_1\boldsymbol{u}_{k+1}. \tag{10}$$

We can prove c) in a similar manner, as has been done for a). ■

For some symmetric–antisymmetric multiwavelet systems, the prefilter can be chosen as the sum/difference prefilter

$$\boldsymbol{R} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

This orthogonal and nonredundant prefilter has been shown to be effective for symmetric–antisymmetric multiwavelet systems (see [9]).

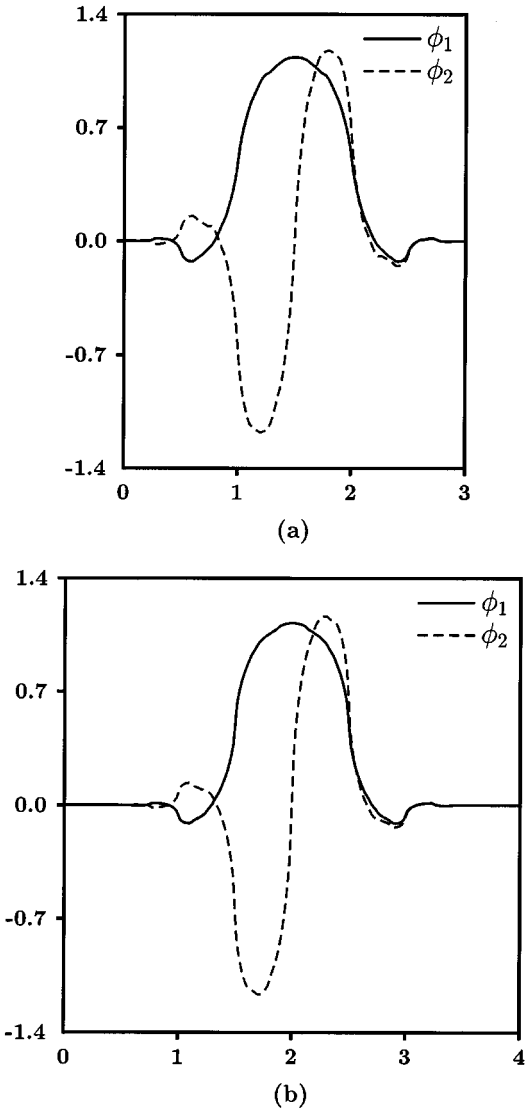

Fig. 1. Plots of multiscaling functions for (a) SA4(1) and (b) SA5(1) multifilters.

We also define two mappings $\mathcal{V}$ and $\mathcal{U}$ that formulate the two vector sequences $\{\boldsymbol{v}_k\}$ and $\{\boldsymbol{u}_k\}$ from a given signal $\{x_k\}$ as

$$\boldsymbol{v}_k = \boldsymbol{R}\begin{bmatrix} x_{2k} \\ x_{2k+1} \end{bmatrix} \quad \text{and } \boldsymbol{u}_k = \boldsymbol{R}\begin{bmatrix} x_{2k-1} \\ x_{2k} \end{bmatrix} \tag{11}$$

respectively. It is easy to check that $\{\boldsymbol{v}_k\}$ and $\{\boldsymbol{u}_k\}$ satisfy (9) and (10). Now, for a signal $\{x_k\}$, we define two new signals $\{\underline{x}_k\}$ and $\{\overline{x}_k\}$ by $\underline{x}_k = x_{k-1}$ and $\overline{x}_k = x_{k+1}$ for any k. The actions of $\mathcal{V}$ and $\mathcal{U}$ on $\{\underline{x}_k\}$ and $\{\overline{x}_k\}$ will produce four vector sequences $\{\underline{\boldsymbol{v}}_k\}$, $\{\underline{\boldsymbol{u}}_k\}$, $\{\overline{\boldsymbol{v}}_k\}$, and $\{\overline{\boldsymbol{u}}_k\}$. It is easy to establish these simple relations: $\underline{\boldsymbol{v}}_k = \boldsymbol{u}_k$, $\underline{\boldsymbol{u}}_k = \boldsymbol{v}_{k-1}$, $\overline{\boldsymbol{v}}_k = \boldsymbol{u}_{k+1}$, and $\overline{\boldsymbol{u}}_k = \boldsymbol{v}_k$. Applying these relations allows us to interpret items a), b), and c) in Theorem 2 with the following equivalent statements.

a) The application of DMWT on an input signal $\{x_k\}$ using $\{\boldsymbol{P}_{e,k}\}_{k=0}^{2N-1}$ and prefiltering with $\mathcal{V}$ will produce the same output as that of using $\{\boldsymbol{P}_{o,k}\}_{k=0}^{2N}$ and prefiltering with $\mathcal{U}$.

b) The application of DMWT on an input signal $\{x_k\}$ using $\{\boldsymbol{P}_{o,k}\}_{k=0}^{2N}$ and prefiltering with $\mathcal{V}$ will produce the same output as the application of DMWT on an input signal $\{\overline{x}_k\}$ using $\{\boldsymbol{P}_{e,k}\}_{k=0}^{2N-1}$ and prefiltering with $\mathcal{V}$.

c) The application of DMWT on an input signal $\{x_k\}$ using $\{\boldsymbol{P}_{e,k}\}_{k=0}^{2N-1}$ and prefiltering with $\mathcal{U}$ will produce the same output as the application of DMWT on an input signal $\{\underline{x}_k\}$ using $\{\boldsymbol{P}_{o,k}\}_{k=0}^{2N}$ and prefiltering with $\mathcal{U}$.

Note that the indices of both even-length and odd-length SAOMF's start from 0. By manipulating the indices of $\boldsymbol{u}$ and $\boldsymbol{v}$ in the right-hand sides of b) and c) in Theorem 2, it is possible to have different starting indices for the even-length and odd-length SAOMF's, and we could rephrase statements b) and c) above as follows.

b)′ The application of DMWT on an input signal $\{x_k\}$ using $\{\boldsymbol{P}_{o,k}\}_{k=0}^{2N}$ and prefiltering with $\mathcal{V}$ will produce the same output as that of using $\{\boldsymbol{P}_{e,k}\}_{k=1}^{2N}$ and prefiltering with $\mathcal{U}$.

c)′ The application of DMWT on an input signal $\{x_k\}$ using $\{\boldsymbol{P}_{e,k}\}_{k=0}^{2N-1}$ and prefiltering with $\mathcal{U}$ will produce the same output as that of using $\{\boldsymbol{P}_{o,k}\}_{k=-1}^{2N-1}$ and prefiltering with $\mathcal{V}$.

V. Conclusions

From the above discussion, we conclude that even-length and odd-length SAOMF's in related pairs are equivalent when the sum/difference prefilter is integrated with the corresponding DMWT. This means that in the absence of code optimization in exploiting the structure of the lowpass/highpass matrix coefficients, it will be more efficient to apply the shorter even-length filter in implementation.

References

[1] C. K. Chui and J. Lian, "A study on orthonormal multiwavelets," *Appl. Numer. Math.*, vol. 20, no. 3, pp. 273–298, 1996.

[2] G. Donovan, J. S. Geronimo, D. P. Hardin, and P. R. Massopust, "Construction of orthogonal wavelets using fractal interpolation functions," *SIAM, J. Math. Anal.*, vol. 27, pp. 1791–1815, 1996.

[3] D. P. Hardin and D. W. Roach, "Multiwavelet prefilters I: Orthogonal prefilters preserving approximation order $p \leq 2$," *IEEE Trans. Circuits Syst. II*, vol. 45, pp. 1106–1112, Aug. 1998.

[4] Q. Jiang, "On the design of multifilter banks and orthonormal multiwavelets bases," *IEEE Trans. Signal Processing*, vol. 46, pp. 3293–3303, Dec. 1998.

[5] J. Lebrun and M. Vetterli, "Balanced multiwavelets theory and design," *IEEE Trans. Signal Processing*, vol. 46, pp. 1119–1124, Apr. 1998.

[6] I. W. Selesnick, "Multiwavelet bases with extra approximation properties," *IEEE Trans. Signal Processing*, vol. 46, pp. 2898–2908, Nov. 1998.

[7] L. Shen, H. H. Tan, and J. Y. Tham, "Symmetric–antisymmetric orthonormal multiwavelets and related scalar wavelets," *Appl. Comput. Harmon. Anal.*, vol. 8, pp. 258–279, May 2000.

[8] V. Strela, P. N. Heller, G. Strang, P. Topiwala, and C. Heil, "The application of multiwavelet filterbanks to image processing," *IEEE Trans. Image Processing*, vol. 8, pp. 548–563, Apr. 1999.

[9] J. Y. Tham, L. Shen, S. L. Lee, and H. H. Tan, "A general approach for analysis and application of discrete multiwavelet transforms," *IEEE Trans. Signal Processing*, vol. 48, pp. 457–464, Feb. 2000.

[10] M. J. Vrhel and A. Aldroubi, "Projection based prefiltering for multiwavelet transforms," *IEEE Trans. Signal Processing*, vol. 46, pp. 3088–3092, Nov. 1998.

[11] C. Weidmann, J. Lebrun, and M. Vetterli, "Significance tree image coding using balanced multiwavelets," in *Proc. Int. Conf. Image Process.*, Chicago, IL, Oct. 1998.

[12] X. G. Xia, J. S. Geronimo, D. P. Hardin, and B. W. Suter, "Design of prefilters for discrete multiwavelet transforms," *IEEE Trans. Signal Processing*, vol. 44, pp. 25–35, Jan. 1996.

[13] X. G. Xia, "A new prefilter design for discrete multiwavelet transforms," *IEEE Trans. Signal Processing*, vol. 46, pp. 1558–1570, June 1998.